GCE214
Applied Mechanics-Statics

Lecture 04: 27/09/2017

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Class: Wednesday (3–5 pm)
Venue: LT1

Etiquettes and MOP

- Attendance is a requirement.
- There may be class assessments, during or after lecture.
- Computational software will be employed in solving problems
- Conceptual understanding will be tested
- Lively discussions are integral part of the lectures.

Lecture content

Rigid Bodies: Equivalent Systems of Forces

- Introduction
- Principle of Transmissibility
- Moment of a Force About a Point
- Rectangular Components of the Moment of a Force
- Moment of a Force about a given Axis

Recommended textbook

- Vector Mechanics for Engineers: Statics and Dynamics by Beer, Johnston, Mazurek, Cornwell. 10th Edition

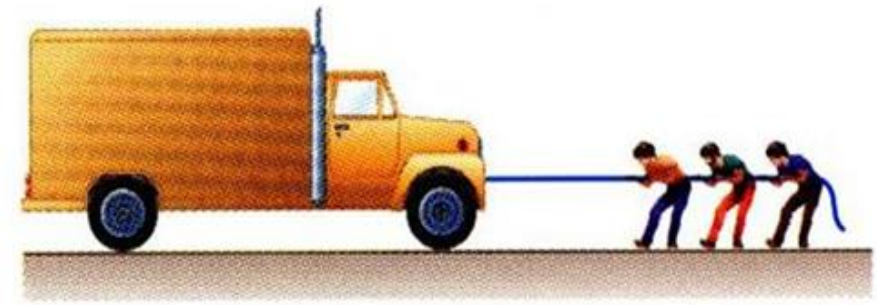
RIGID BODIES: EQUIVALENT SYSTEMS OF FORCES

INTRODUCTION

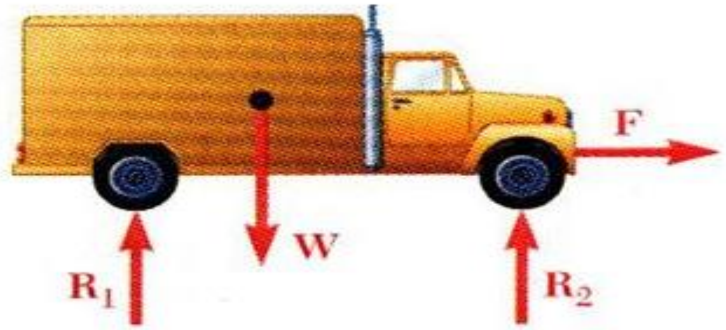
- Treating a body as a single particle is not always possible. In most cases the size of the body and the specific points of application of the forces must be considered.
- In elementary mechanics most bodies are assumed to be ***rigid***, in other words, the actual deformations are small and don't affect the equilibrium conditions or the motion of the body.
- Our study will focus on the effect forces exerted on rigid bodies and how to replace a given system of forces with a simpler equivalent system.
    - Moment of a force about a point
    - Moment of a force about an axis
    - Moment due to a couple
- Any system of forces acting on a rigid body can be replaced by an equivalent system consisting of one force acting at a given point and one couple

INTRODUCTION: EXTERNAL AND INTERNAL FORCES

- Forces acting on a rigid body are mainly
  (1) External forces, and
  (2) Internal forces.



- An example external forces acting on a rigid body is shown in a free-body diagram.



- If unopposed, each external force can impart a motion of translation or rotation, or both.

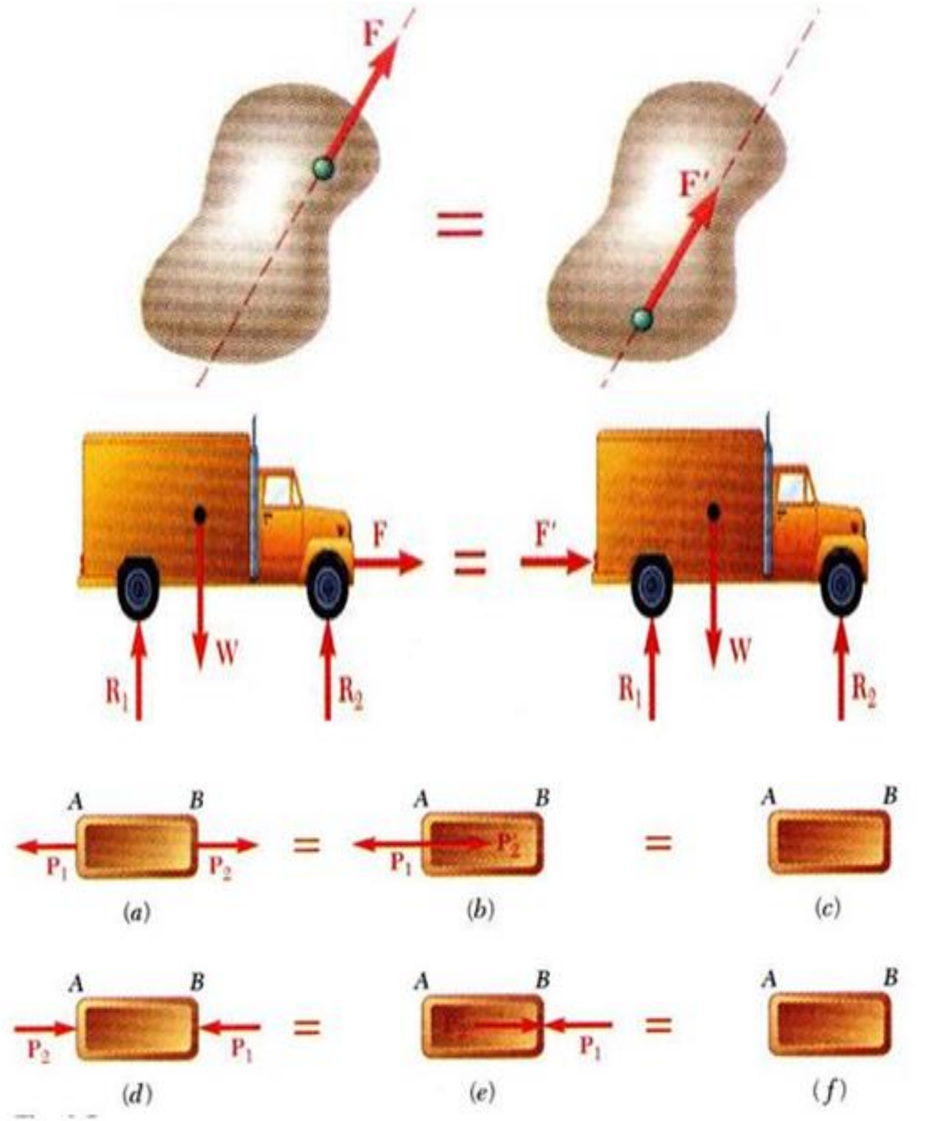
PRINCIPLE OF TRANSMISSIBILITY

- ***Principle of*** ***transmissibility-*** Conditions of equilibrium or motion are unaffected by transmitting a force along its line of action

  NOTE: $\boldsymbol{F}$ and $\boldsymbol{F}'$ are equivalent forces

- Moving the point of application of the force $\boldsymbol{F}$ to the rear bumper does not affect the motion or the other forces acting on the truck.

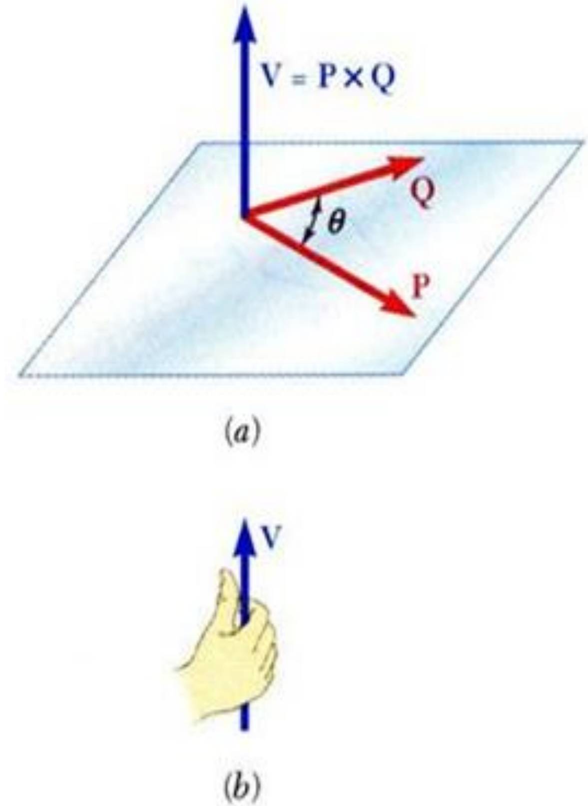
- If unopposed, each external force can impart a motion of translation or rotation, or both.



VECTOR PRODUCT OF TWO VECTORS

- Concept of the ***moment of a force about a point*** is more easily understood through applications of the vector or cross product

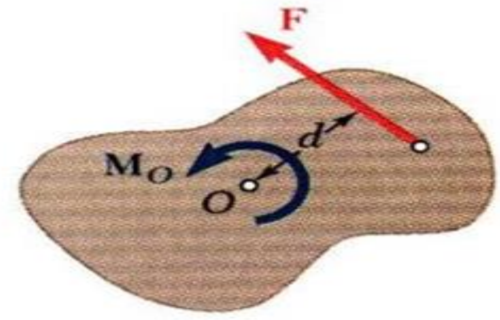
- Vector product of two vectors $\boldsymbol{P}$ and $\boldsymbol{Q}$ is defined as the vector $\boldsymbol{V}$ which satisfies the following conditions:
  1. Line of action $\boldsymbol{V}$ is perpendicular to the plane containing $\boldsymbol{P}$ and $\boldsymbol{Q}$.
  2. Magnitude of $\boldsymbol{V}$ is $V = PQ\sin\theta$
  3. Direction of $\boldsymbol{V}$ is obtained from the right-hand rule. An example external forces acting on a rigid body is shown in a free-body diagram.



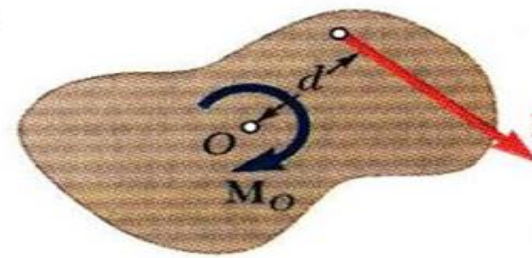
- Vector products:
  - are not commutative, $\boldsymbol{Q} \times \boldsymbol{P} = -(\boldsymbol{P} \times \boldsymbol{Q})$
  - are distributive, $\boldsymbol{P} \times (\boldsymbol{Q_1} + \boldsymbol{Q_2}) = \mathbf{P} \times \boldsymbol{Q_1} + \boldsymbol{P} \times \boldsymbol{Q_2}$
  - are associative, $(\boldsymbol{P} \times \boldsymbol{Q}) \times \boldsymbol{S} = \boldsymbol{P} \times (\boldsymbol{Q} \times \boldsymbol{S})$

MOMENT OF A FORCE ABOUT A POINT

- Two-dimensional structures have length and breadth but negligible depth and are subjected to forces contained in the plane of the structure.
- The plane of the structure contains the point O and the force F. M_o the moment of the force about O is perpendicular to the plane.
- If the force tends to rotate the structure counterclockwise, the sense of the moment vector is out of the plane of the structure and the magnitude of the moment is positive.
- If the force tends to rotate the structure clockwise, the sense of the moment vector is into the plane of the structure and the magnitude of the moment is negative.



(a) $M_O = +Fd$



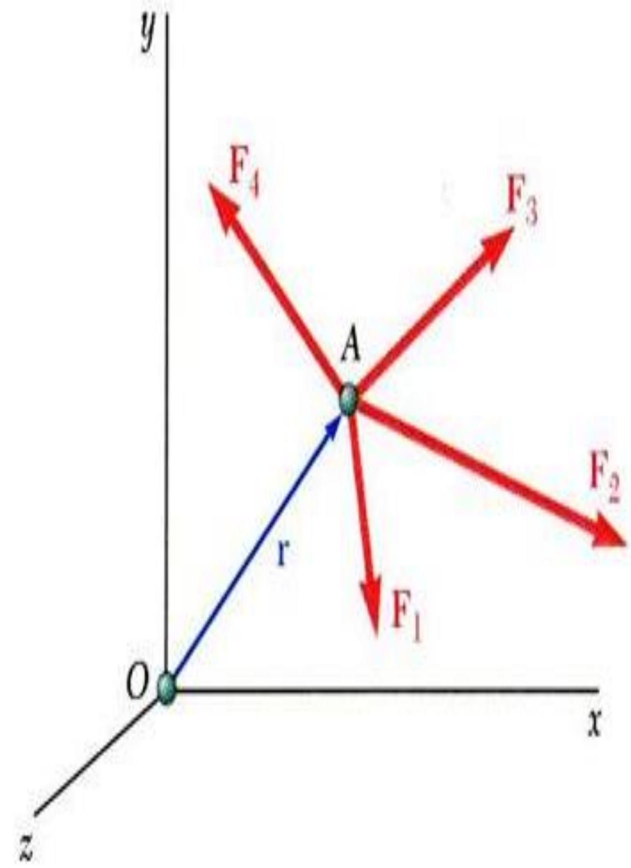
(b) $M_O = -Fd$

VARIGON'S THEOREM

- The moment about a given point O of the resultant of several concurrent forces is equal to the sum of he moments of the various moments about the same point O.

- $\boldsymbol{r} \times (\boldsymbol{F_1} + \boldsymbol{F_2} + \cdots) = \boldsymbol{r} \times \boldsymbol{F_1} + \boldsymbol{r} \times \boldsymbol{F_2} + \cdots$

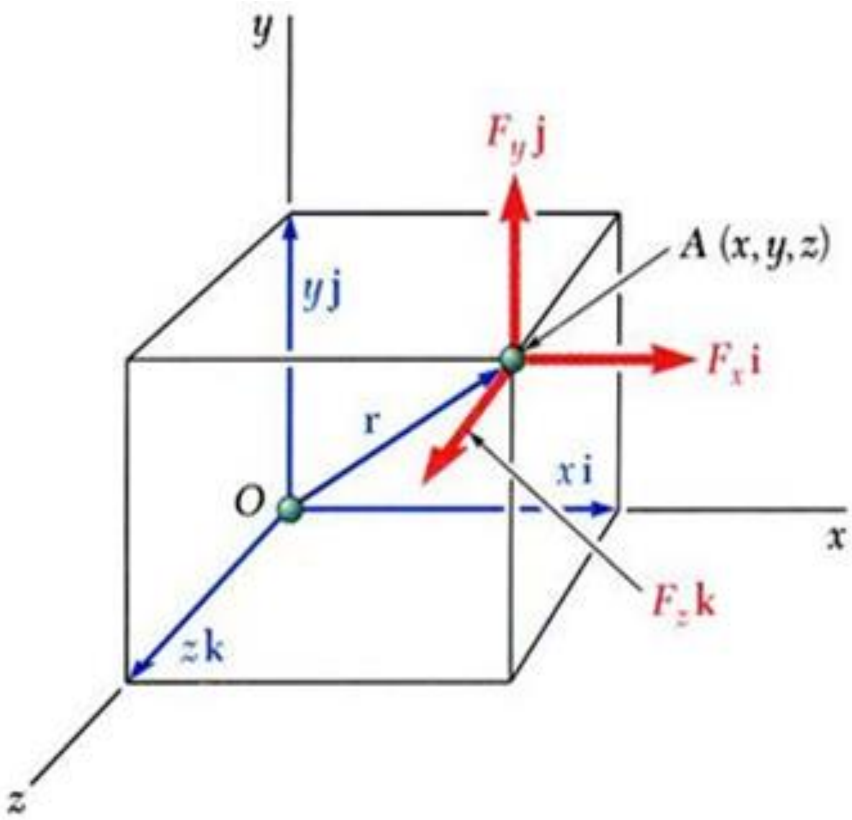
- Varigon's Theorem makes it possible to replace the direct determination of the moment of a force ***F*** by the moments of two or more components forces of ***F***



RECTANGULAR COMPONENTS OF THE MOMENT OF A FORCE

- The moment of $\boldsymbol{F}$ about O

$$\boldsymbol{M}_o = \boldsymbol{r} \times \boldsymbol{F},$$
$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$
$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$



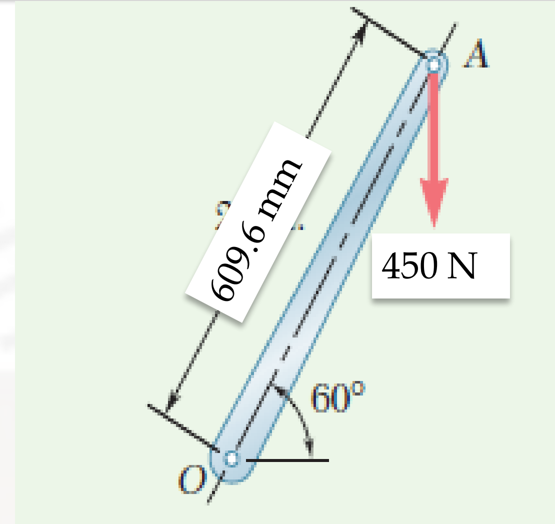
$$\boldsymbol{M}_o = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

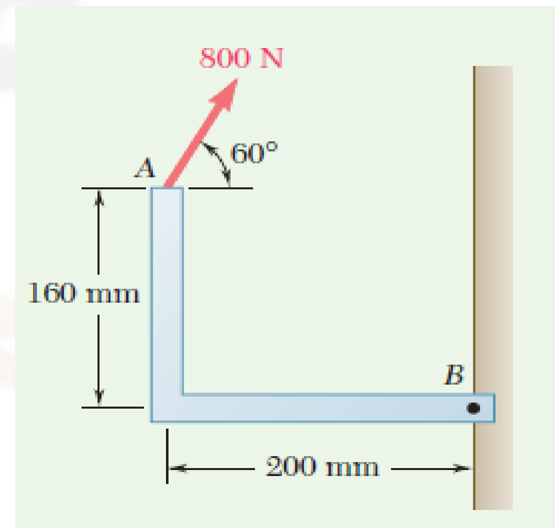
$$= \left(yF_z - zF_y\right)\boldsymbol{i} + (zF_x - xF_z)\boldsymbol{J} + \left(xF_y - yF_x\right)\boldsymbol{k}$$

EXAMPLES

1. A 450 N vertical force is applied to the end of a lever which is attached to a shaft at *O*. Determine (*a*) the moment of the 450 N force about *O*; (*b*) the horizontal force applied at *A* which creates the same moment about *O*; (*c*) the smallest force applied at *A* which creates the same moment about *O*; (*d*) how far from the shaft a 1080 N vertical force must act to create the same moment about *O*; (*e*) whether any one of the forces obtained in parts *b*, *c*, and *d* is equivalent to the original force.



2. A force of 800 N acts on a bracket as shown. Determine the moment of the force about *B*.

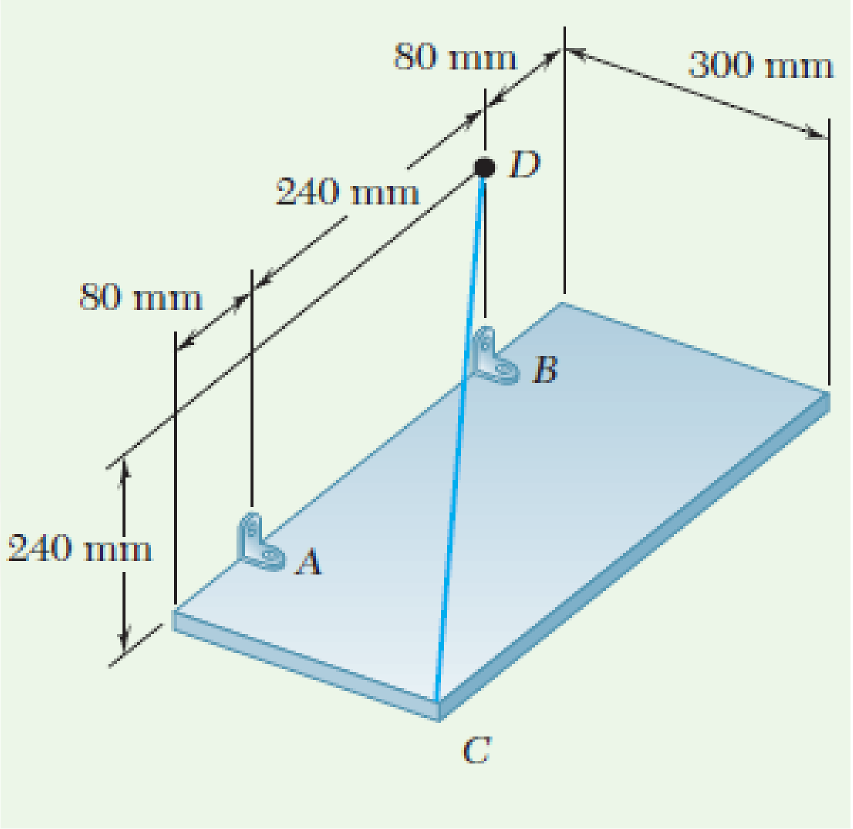


CLASSWORK

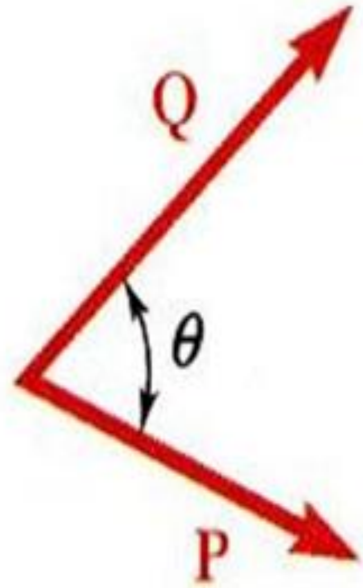
Treat Q.2 with the force of 800 N acting at 120° to the horizontal on the bracket. Determine the moment of the force about *B*.

EXAMPLES

3. A rectangular plate is supported by brackets at *A* and *B* and by a wire *CD*. Knowing that the tension in the wire is 200 N, determine the moment about 80 mm *A* of the force exerted by the wire on point *C* .



SCALAR PRODUCT OF TWO VECTORS



- The scalar product or dot product between two vectors **P** and **Q** is defined as

  $\boldsymbol{P} \bullet \boldsymbol{Q} = PQ\cos\theta$ (scalar result)

- Scalar products:
  - are commutative, $\boldsymbol{P} \bullet \boldsymbol{Q} = \boldsymbol{Q} \bullet \boldsymbol{P}$
  - are distributive, $\boldsymbol{P} \bullet (\boldsymbol{Q}_1 + \boldsymbol{Q}_2) = \boldsymbol{P} \bullet \boldsymbol{Q}_1 + \boldsymbol{P} \bullet \boldsymbol{Q}_2$
  - are not associative, $(\boldsymbol{P} \bullet \boldsymbol{Q}) \bullet \boldsymbol{S} = undefined$

- Scalar products with Cartesian unit components,

$$\boldsymbol{P} \bullet \boldsymbol{Q} = \left(P_x\boldsymbol{i} + P_y\boldsymbol{j} + P_z\boldsymbol{k}\right) \bullet \left(Q_x\boldsymbol{i} + Q_y\boldsymbol{j} + Q_z\boldsymbol{k}\right)$$

$$\boldsymbol{i} \bullet \boldsymbol{i} = 1, \quad \boldsymbol{j} \bullet \boldsymbol{j} = 1, \quad \boldsymbol{k} \bullet \boldsymbol{k} = 1, \quad \boldsymbol{i} \bullet \boldsymbol{j} = 0, \quad \boldsymbol{j} \bullet \boldsymbol{k} = 0, \quad \boldsymbol{k} \bullet \boldsymbol{i} = 0$$

$$\boldsymbol{P} \bullet \boldsymbol{Q} = P_xQ_x + P_yQ_y + P_zQ_z$$

$$\boldsymbol{P} \bullet \boldsymbol{P} = P_x^2 + P_y^2 + P_z^2$$

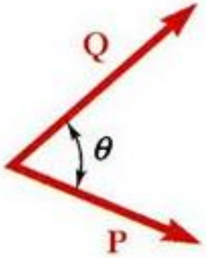
RIGID BODIES: EQUIVALENT SYSTEMS OF FORCES

SCALAR PRODUCT OF TWO VECTORS: APPLICATIONS

- Angle between two vectors:

$$\bar{P} \bullet \bar{Q} = PQ\cos\theta = P_x Q_x + P_y Q_y + P_z Q_z$$

$$\cos\theta = \frac{P_x Q_x + P_y Q_y + P_z Q_z}{PQ}$$

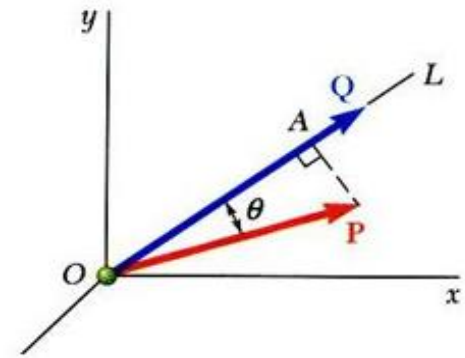


- Projection of a vector on a given axis:

$$P_{OL} = P\cos\theta = \text{projection of } P \text{ along } OL$$

$$\bar{P} \bullet \bar{Q} = PQ\cos\theta$$

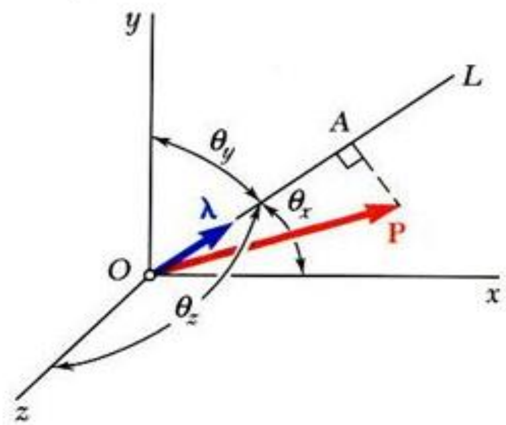
$$\frac{\bar{P} \bullet \bar{Q}}{Q} = P\cos\theta = P_{OL}$$



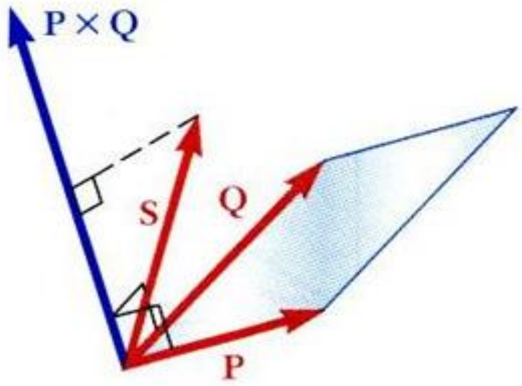
- For an axis defined by a unit vector:

$$P_{OL} = \bar{P} \bullet \bar{\lambda}$$

$$= P_x\cos\theta_x + P_y\cos\theta_y + P_z\cos\theta_z$$



MIXED TRIPLE PRODUCT OF THREE VECTORS



- Mixed triple product of three vectors,

$$\bar{S}\bullet\left(\bar{P}\times\bar{Q}\right)=\text{scalar result}$$

- The six mixed triple products formed from ***S***, ***P***, and ***Q*** have equal magnitudes but not the same sign,

$$\bar{S}\bullet\left(\bar{P}\times\bar{Q}\right)=\bar{P}\bullet\left(\bar{Q}\times\bar{S}\right)=\bar{Q}\bullet\left(\bar{S}\times\bar{P}\right)$$
$$=-\bar{S}\bullet\left(\bar{Q}\times P\right)=-\bar{P}\bullet\left(\bar{S}\times\bar{Q}\right)=-\bar{Q}\bullet\left(\bar{P}\times\bar{S}\right)$$

- Evaluating the mixed triple product,

$$\bar{S}\bullet\left(\bar{P}\times\bar{Q}\right)=S_x\left(P_yQ_z-P_zQ_y\right)+S_y\left(P_zQ_x-P_xQ_z\right)$$
$$+S_z\left(P_xQ_y-P_yQ_x\right)$$
$$=\begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

MOMENT OF A ABOUT A GIVEN AXIS

- Moment $\boldsymbol{M_O}$ of a force $\boldsymbol{F}$ applied at the point $\boldsymbol{A}$ about a point $\boldsymbol{O}$,

$$\bar{M}_O = \bar{r} \times \bar{F}$$

- Scalar moment M_{OL} about an axis $\boldsymbol{OL}$ is the projection of the moment vector $\boldsymbol{M_O}$ onto the axis,

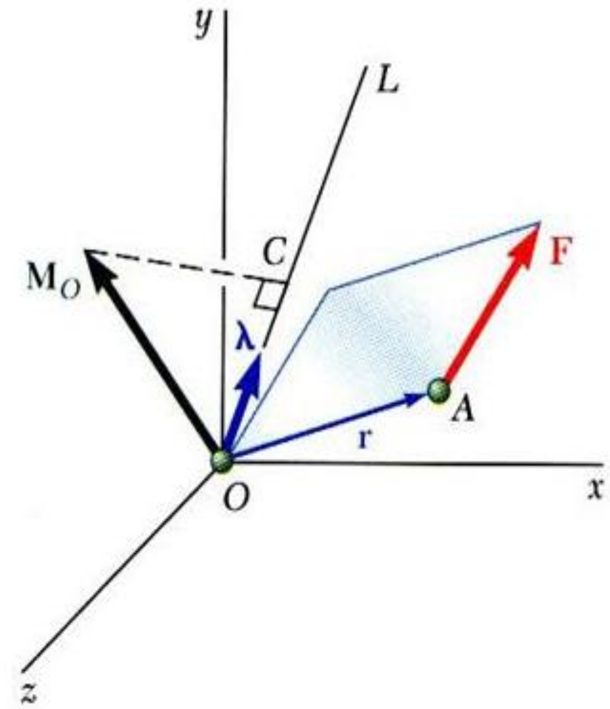
$$M_{OL} = \bar{\lambda} \bullet \bar{M}_O = \bar{\lambda} \bullet (\bar{r} \times \bar{F})$$

- Moments of $\boldsymbol{F}$ about the coordinate axes,

$$M_x = yF_z - zF_y$$

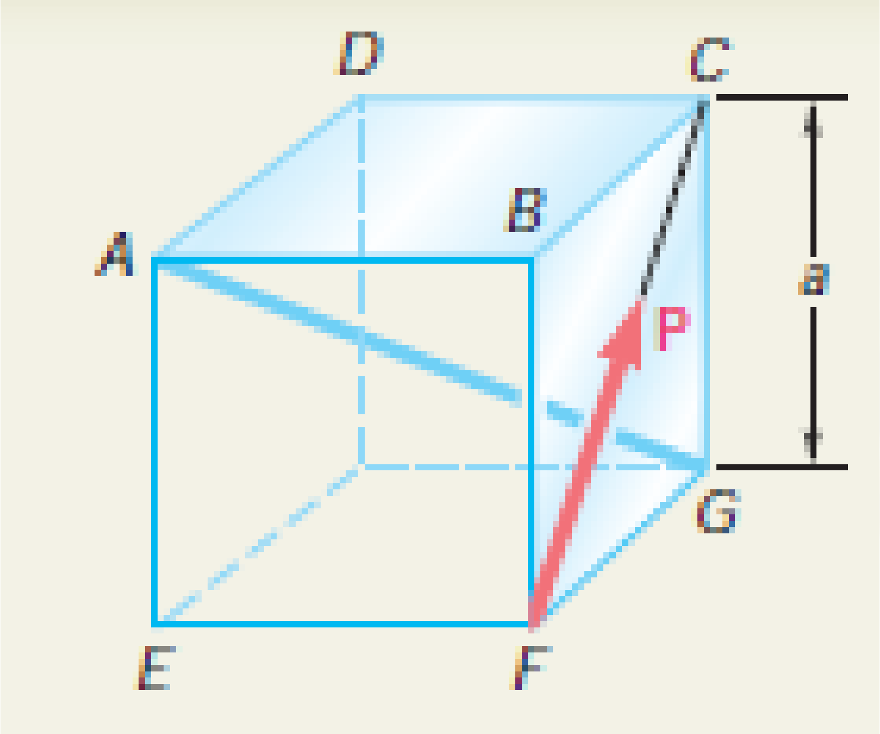
$$M_y = zF_x - xF_z$$

$$M_z = xF_y - yF_x$$

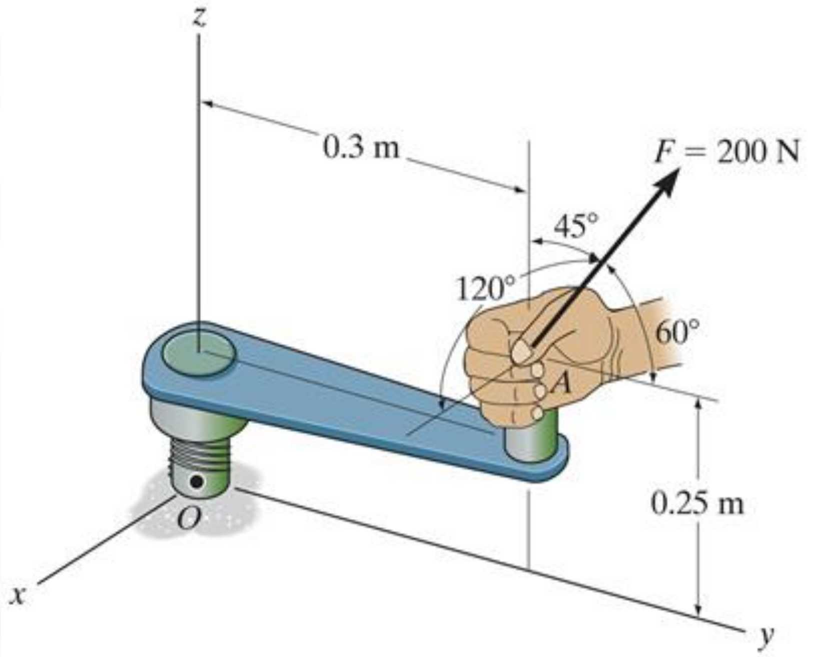


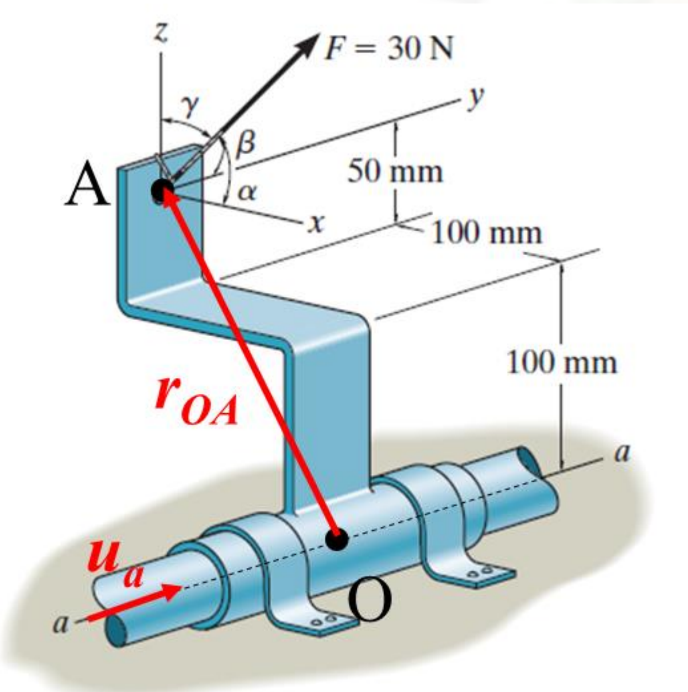
EXAMPLES

4. A cube of side *a* is acted upon by a force ***P*** as shown. Determine the moment of ***P*** (*a*) about *A* , (*b*) about the edge *AB* , (*c*) about the diagonal *AG* of the cube, (*d*). Using the result of part *c* , determine the perpendicular distance between *AG* and *FC*. *AB*



5. Find the magnitude of the moment of the force about the x-axis for a force 200 N acting as shown in the figure.





6. The force F acts on a bracket as shown in the figure. Take $\alpha = 60°$, $\beta = 60°$, $\gamma = 45°$. Find the magnitude of the moment about a-a axis.

Principle of Transmissibility

The ***principle of*** ***transmissibility*** states that the conditions of equilibrium or motion of a rigid body will remain unchanged if a force ***F*** acting at a given point of the rigid body is replaced by a force ***F′*** of the same magnitude and same direction, but acting at a different point, *provided that the two forces have the same line of action*